Subject CM2

CMP Upgrade 2021/22

CMP Upgrade

This CMP Upgrade lists the changes to the Syllabus objectives, Core Reading and the ActEd material since last year that might realistically affect your chance of success in the exam. It is produced so that you can manually amend your 2021 CMP to make it suitable for study for the 2022 exams. It includes replacement pages and additional pages where appropriate. Alternatively, you can buy a full set of up-to-date Course Notes / CMP at a significantly reduced price if you have previously bought the full-price Course Notes / CMP in this subject. Please see our 2022 *Student Brochure* for more details.

This CMP Upgrade contains:

- all significant changes to the Syllabus objectives and Core Reading
- additional changes to the ActEd Course Notes and Assignments that will make them suitable for study for the 2022 exams.

1 Changes to the Syllabus objectives

There have been no ***non-trivial*** changes to the Syllabus objectives.

2 Changes to the Core Reading

This section contains all the ***non-trivial*** changes to the Core Reading.

Chapter 3

Page 12

A new section with examples of behavioural finance issues in retail financial services has been added after the Loss aversion and Endowment effects. It reads:

These types of behaviour can be seen in retail financial services. For example:

- **Reference dependence: Retail banking products may be offered to new customers with preferential terms for initial periods, such as higher interest rates on deposits or lower interest rates on credit cards or mortgages. Customers may be attracted by the initially favourable terms and loss sight of the value of the products over their expected lives.**
- **Loss aversion: Customers may be willing to pay high prices for insurance products because they get 'peace of mind' by avoiding potential losses. In the UK, large numbers of customers bought personal protection insurance (PPI), which covered repayment of loans such as mortgages and credit cards, in the event that the borrower died, became ill or disabled or become unemployed. In some cases, the regulator found that PPI had been 'mis-sold' and banks had to compensate their customers.**
- **Endowment effects: Many customers hold a current account for decades, without considering whether alternative products might offer better value. More generally, the level of switching in retail banking product markets is low, despite the availability of price comparison websites which make it easy for customers to compare products and to switch between providers and products.**

Chapter 4

Page 16

A new Core Reading paragraph has been added. It reads:

In the banking crisis of 2007-08, the importance of 'tail risk' was underlined by the extreme losses suffered by some banks, particularly on complex securities based on US sub-prime mortgages. Since then, the banking regulations (Basel III) have replaced VaR measures with measures of expected shortfall. For any given percentile, measures of expected shortfall will be higher than VaR measures, particularly if the distribution is fat-tailed.

Chapter 18

Page 20

Some Core Reading on the CIR model has been added. Replacement pages are provided so that you have a new Section 3.3.

Chapter 19

Page 5

Section 2 is new. Replacement pages are provided.

3 Changes to the ActEd material

Chapter 4

Page 16

Some ActEd text and an example has been added to accompany the new Core Reading paragraph. Replacement pages are provided.

Chapter 18

Page 20

Section 3.3 has been reworded and reordered to accommodate the new Core Reading on the CIR model. Replacement pages are provided.

Page 30

Section 5 has been removed from the course.

Chapter 19

Page 5

Section 2 is new. Replacement pages are provided.

4 Changes to the X Assignments

The X Assignments have been changed significantly to reflect the online nature of the exams. We have not detailed all of the changes in this upgrade.

If you would like the new assignments *without* marking, then retakers can purchase an updated CMP or standalone X Assignments at a significantly reduced price. Further information on retaker discounts can be found at:

www.acted.co.uk/paper_reduced_prices.html

If you wish to submit your scripts for marking but have only an old version, then you can order the current assignments free of charge if you have purchased the same assignments in the same subject in a previous year and have purchased marking for the 2022 session. We only accept the current version of assignments for marking, *ie* those published for the sessions leading to the 2022 exams.

5 Changes to the Y Assignments

There have been no ***non-trivial*** changes to the Y Assignments.

6 Other tuition services

In addition to the CMP you might find the following services helpful with your study.

6.1 Study material

We also offer the following study material in Subject CM2:

- Flashcards
- Revision Notes
- ASET (ActEd Solutions with Exam Technique) and Mini-ASET
- Mock Exam and AMP (Additional Mock Pack).

For further details on ActEd's study materials, please refer to the 2022 *Student Brochure*, which is available from the ActEd website at **www.ActEd.co.uk**.

6.2 Tutorials

We offer the following (face-to-face and/or online) tutorials in Subject CM2:

- a set of Regular Tutorials (lasting four full days)
- a Block (or Split Block) Tutorial (lasting four full days)
- an Online Classroom.

For further details on ActEd's tutorials, please refer to our latest *Tuition Bulletin*, which is available from the ActEd website at **www.ActEd.co.uk**.

6.3 Marking

You can have your attempts at any of our assignments or mock exams marked by ActEd. When marking your scripts, we aim to provide specific advice to improve your chances of success in the exam and to return your scripts as quickly as possible.

For further details on ActEd's marking services, please refer to the 2022 *Student Brochure*, which is available from the ActEd website at **www.ActEd.co.uk**.

6.4 Feedback on the study material

ActEd is always pleased to get feedback from students about any aspect of our study programmes. Please let us know if you have any specific comments or general suggestions about how we can improve the study material. We will incorporate as many of your suggestions as we can when we update the course material each year.

If you have any comments on this course please send them by email to **CM2@bpp.com**.

Example

Consider two investments X and Y, with returns in £m of $25-R_X$ and $25-R_Y$, where $R_X \sim \exp(0.05)$ and $R_Y \sim Pareto(3.5,44)$.

Both investments have very similar Values at Risk at the 5% significance level:

- the VaR for the first investment, involving X, is $59.91465-25=£34.91m$, since $F_X(59.91465)=0.95$, and
- the VaR for the second investment, involving Y, is $59.55606-25=£34.56m$, since $F_Y(59.55606)=0.95$.

You can check these using the distribution functions on pages 11 and 14 of the Tables.

Conditional TailVaR is an expected shortfall measure that shows us the extra expected loss on top of the VaR if we are in the 5% tail of worst outcomes.

The conditional TailVaR for X is:

$$\int_{59.91465}^{\infty} (x-59.91465)0.05e^{-0.05x}dx \Big/ 0.05 = \cdots = £20.00m$$

The conditional TailVaR for Y is:

$$\int_{59.55606}^{\infty} (y-59.55606)\frac{3.5\times 44^{3.5}}{(44+y)^{4.5}}dy \Big/ 0.05 = \cdots = £44.42m$$

If you are keen to practise your integration skills, you can again check these if you like. The relevant probability density functions are given on pages 11 and 14 of the Tables.

So, loosely speaking, the 5th worst outcome out of 100 outcomes for both investments is that they lose about £35m. However:

- For X, the total expected loss if we are in the 5% tail of worst outcomes is $34.91+20=£54.91m$, a much higher number than £35m.
- For Y, the total expected loss if we are in the 5% tail of worst outcomes is $34.56+44.42=£78.98m$, also a much higher number than £35m.

Because the Pareto distribution has a heavier tail than the exponential distribution, the expected loss is far greater for the investment with return $25-R_Y$.

2 Relationship between risk measures and utility functions

An investor using a particular risk measure will base their decisions on a consideration of the available combinations of risk and expected return. Given a knowledge of how this trade-off is made it is possible, in principle, to construct the investor's underlying utility function. Conversely, given a particular utility function, the appropriate risk measure can be determined.

For example, if an investor has a quadratic utility function, the function to be maximised in applying the expected utility theorem will involve a linear combination of the first two moments of the distribution of return.

In other words, if an investor has a quadratic utility function then their attitude towards risk and return can be expressed purely in terms of the mean and variance of investment opportunities.

Thus variance of return is an appropriate measure of risk in this case.

Question

(i) State the expected utility theorem.

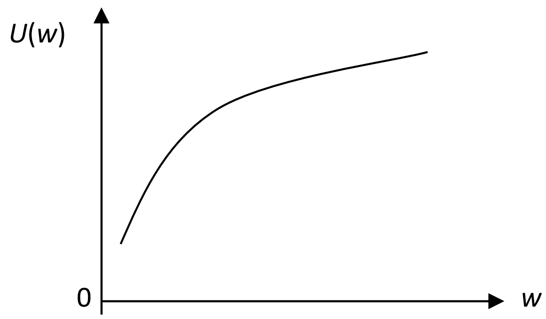
(ii) Draw a typical utility function for a non-satiated, risk-averse investor.

Solution

(i) The *expected utility theorem* states that:

- a function, $U(w)$, can be constructed representing an investor's utility of wealth, w
- the investor faced with uncertainty makes decisions on the basis of maximising the *expected value* of utility.

(ii)



Non-satiated investors prefer more wealth to less and so the graph slopes upwards, *ie* $U'(w) > 0$.

Risk-averse investors have diminishing marginal utility of wealth and so the slope of the graph decreases with w, *ie* $U''(w) < 0$.

By use of the chain rule, and noting that $\dfrac{\partial \tau}{\partial T}=1$, this gives:

$$f(t,T)=-\frac{\partial}{\partial T}[a(\tau)-b(\tau)r(t)]$$

$$=-\frac{\partial \tau}{\partial \tau}\frac{\partial}{\partial T}[a(\tau)-b(\tau)r(t)]$$

$$=-\frac{\partial \tau}{\partial T}\frac{\partial}{\partial \tau}[a(\tau)-b(\tau)r(t)]=-a'(\tau)+b'(\tau)r(t)$$

From the definitions of $b(\tau)$ and $a(\tau)$, we find that:

$$b'(\tau)=\frac{d}{d\tau}\left(\frac{1-e^{-\alpha\tau}}{\alpha}\right)=e^{-\alpha\tau}$$

and:

$$a'(\tau)=\frac{d}{d\tau}\left[(b(\tau)-\tau)\left(\mu-\frac{\sigma^2}{2\alpha^2}\right)-\frac{\sigma^2}{4\alpha}b(\tau)^2\right]$$

$$=(b'(\tau)-1)\left(\mu-\frac{\sigma^2}{2\alpha^2}\right)-\frac{\sigma^2}{4\alpha}\times 2b(\tau)b'(\tau)$$

$$=(e^{-\alpha\tau}-1)\left(\mu-\frac{\sigma^2}{2\alpha^2}\right)-\frac{\sigma^2}{2\alpha}\left(\frac{1-e^{-\alpha\tau}}{\alpha}\right)e^{-\alpha\tau}$$

$$=-(1-e^{-\alpha\tau})\left(\mu-\frac{\sigma^2}{2\alpha^2}\right)-\frac{\sigma^2}{2\alpha^2}(1-e^{-\alpha\tau})e^{-\alpha\tau}$$

Substituting these expressions into the general formula for $f(t,T)$ gives the required answer.

Question

Write down an expression in terms of the model parameters for the long rate, *ie* the instantaneous forward rate corresponding to $T-t=\infty$, according to the Vasicek model.

Solution

Letting $T\to\infty$ (and hence $\tau\to\infty$) in the equation for $f(t,T)$ gives:

$$f(t,\infty)=r(t)\times 0+\left(\mu-\frac{\sigma^2}{2\alpha^2}\right)(1-0)+\frac{\sigma^2}{2\alpha^2}(1-0)\times 0=\mu-\frac{\sigma^2}{2\alpha^2}$$

The curves shown on the graph of gilt yields in Section 1 were fitted using a Vasicek model with parameter values $\alpha = 0.131$, $\mu = 0.083$ and $\sigma = 0.037$.

Question

'The particular model used for the graph implies that interest rates are mean-reverting to the value $\mu = 0.083$.'

True or false?

Solution

The dynamics of $r(t)$ for this particular Vasicek model are:

$$dr(t) = -0.131\,[r(t) - 0.083]dt + 0.037 d\tilde{W}(t)$$

under the *risk-neutral* probability measure Q. Under this measure $\tilde{W}(t)$ is standard Brownian motion and therefore has zero drift and the process mean-reverts to the value 0.083.

However, under the *real-world* probability measure P, $\tilde{W}(t)$ would have non-zero drift and the process will mean-revert to a different value. In fact, although we will not prove it here, the long-term rate in the real world can be found from the formula derived in the previous question, namely:

$$\mu - \frac{\sigma^2}{2\alpha^2} = 0.0431 \quad ie\ \ 4.31\%$$

3.3 The Cox-Ingersoll-Ross (CIR) model (1985)

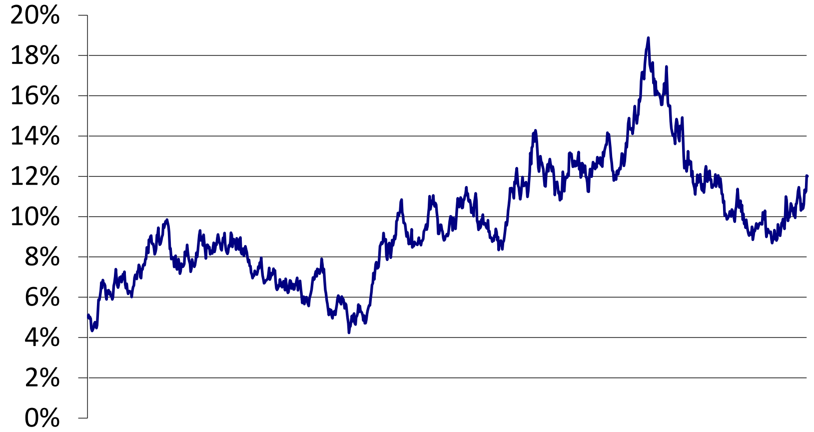
In Vasicek's model (and Hull-White, below) interest rates are not strictly positive. This assumption is not ideal for a short-rate model. CIR use the Feller, or square root mean reverting process, which is positive (it can instantaneously touch 0 but immediately rebounds):

$$\boldsymbol{dr_t = \alpha(\mu - r_t)dt + \sigma\sqrt{r_t}\, d\tilde{W}_t}$$

for constants $\alpha > 0$, $\mu > 0$ and, σ.

We can see that the form of the drift of r_t is the same as for the Vasicek model. The critical difference between the two models occurs in the volatility, which is increasing in line with the square root of r_t. Since this diminishes to zero as r_t approaches zero, and provided σ^2 is not too large ($\sigma^2 \le 2\alpha\mu$), we can guarantee that r_t will not hit zero. Consequently all other interest rates will also remain strictly positive.

The graph below shows a simulation of this process based on the parameter values $\alpha = 0.1$, $\mu = 0.06$ and $\sigma = 0.1$.



Simulation from Cox-Ingersoll-Ross model

It is not possible to solve the SDE for the CIR model.

The associated PDE is:

$$\frac{\partial g(t,r_t)}{\partial t} + \frac{\partial g(t,r_t)}{\partial r_t}\alpha(\mu - r_t) + \frac{1}{2}\sigma^2 r_t^2 \frac{\partial^2 g(t,r_t)}{\partial r_t^2} - r_t\, g(t,r_t) = 0$$

and, again, $P(t,T) = e^{a(\tau) - b(\tau) r_t}$ **with:**

$$b(\tau) = \frac{2\left(e^{\gamma\tau} - 1\right)}{(\gamma + \alpha)\left(e^{\gamma\tau} - 1\right) + 2\gamma}$$

$$a(\tau) = \frac{2\alpha\mu}{\sigma^2} \ln\left(\frac{2\gamma e^{\frac{1}{2}(\gamma+\alpha)\tau}}{(\gamma + \alpha)\left(e^{\gamma\tau} - 1\right) + 2\gamma}\right)$$

where $\gamma = \sqrt{\alpha^2 + 2\sigma^2}$ **.**

It turns out that these values for a **and** b **are not that different from those in Vasicek's model.**

The distribution of r_t **is given by a 'non-central chi-squared' distribution. this is a 'fat-tailed' distribution.**

If $X_1, X_2, \ldots, X_n$ are independent random variables, each with a $N(0,1)$ distribution, then $Y = X_1^2 + X_2^2 + \cdots + X_n^2$ has a chi-square distribution with n degrees of freedom.

If $X_1, X_2, \ldots, X_n$ are independent random variables and $X_i \sim N(d_i, 1)$, then $Y_d = X_1^2 + X_2^2 + \cdots + X_n^2$ is said to have a *non-central* chi-squared distribution with n degrees of freedom and non-centrality parameter $d = \sum_{i=1}^{n} d_i^2$.

So the non-central chi-squared distribution can be thought of as a lopsided version of the ordinary chi-square distribution.

Question

What is the mean of the non-central chi-squared distribution with n degrees of freedom and non-centrality parameter d ?

Solution

Since $X_i \sim N(d_i, 1)$, we find that:

$$E[X_i^2] = Var(X_i) + [E(X_i)]^2 = 1 + d_i^2$$

It follows that:

$$E[Y_d] = E[X_1^2 + X_2^2 + \cdots + X_n^2] = \sum_{i=1}^{n} (1 + d_i^2) = n + d$$

3.4 Vasicek and CIR yield curves

Recall:

$$\boldsymbol{r(t,T) = -\frac{\ln P(t,T)}{T-t}}$$

and:

$$P(t,T) = e^{a(t,T) - b(t,T) r_t}$$

so we have:

$$\boldsymbol{r(t,T) = -\frac{a(t,T) - b(t,T) r_t}{T-t}}$$

Alternatively:

$$r(\tau) = -\frac{a(\tau) - b(\tau) r_t}{\tau}$$

2 Provisions for banking expected credit losses (ECL)

In this section we use an example to show how to calculate the provisions for banking expected credit losses.

ABC Bank has recently made:

- **1,000 loans,**
- **each for £1,000, to small businesses.**

These loans are repayable in 5 years.

Banks typically hold provisions for expected credit losses, in the way that insurance companies hold reserves for claims. In determining its provisions, ABC Bank:

- **discounts its expected credit losses,**
- **which will occur at the end of each year, at 5%.**

ABC Bank wishes to hold a provision at the beginning of each year for its potential loss over the next 12 months.

Taking into consideration the level of credit risk associated with loans to small businesses and the favourable economic outlook for the next 12 months, ABC Bank estimates that:

- **the loans have a probability of default (PD) of 0.4% over the next 12 months**
- **in the event of default, it will only be able to recover 70% of the loans, *ie*, the loss given default will be 30%.**

Question

Calculate the following:

(i) The probability that there will not be a default in the next 12 months.

(ii) The provision that the bank should hold for these loans, based on its expected credit losses at the end of 12 months.

Solution

(i) The probability that there will not be a default in the next 12 months is given by:

$$1 - PD = 1 - 0.4\% = 0.996$$

(ii) Expected credit losses at the end of 12 months are:

$$ECL = EAD \times PD \times LGD$$

(Expected Credit Loss = Exposure At Default × Probability of Default × Loss Given Default)

$$\Rightarrow \quad ECL = £1{,}000{,}000 \times 0.4\% \times 30\% = £1{,}200.00$$

ABC Bank should hold a provision amounting to $\dfrac{ECL}{1.05} = \dfrac{£1{,}200.00}{1.05} = £1{,}142.86$ **.**

Almost immediately after the loans have been made, an external shock has occurred and it is now forecast that the economy will move into recession in a year. In past periods of recession, credit losses have typically been much higher than in periods of economic growth, and periods of recession have typically lasted for two years.

Given the worse and more uncertain economic outlook, ABC Bank now wishes to take a more prudent approach by holding at the outset a provision that allows for expected credit losses on the portfolio of small business loans over the 5-year life of the loans.

Based on experience of this type of loan in previous recessions, ABC now estimates that:

- **in years 2 and 3, the probability of default will be three times as high, at 1.2%**
- **in the event of default, depressed values of property and other assets will increase the loss given default to 50%.**

That is, the bank's forecasts for each of the five years is as follows:

Year	**1**	**2**	**3**	**4**	**5**
PD	**0.4%**	**1.2%**	**1.2%**	**0.4%**	**0.4%**
LGD	**30%**	**50%**	**50%**	**30%**	**30%**

Question

Calculate the provision that the bank should hold for these loans, based on its expected credit losses over the 5-year period of the loans.

Solution

ABC Bank should hold a provision amounting to:

$$\sum_{i=1}^{i=5} \frac{£1{,}000{,}000 \times PD_i \times LGD_i}{(1+r)^i}$$

Year	**EAD**	**PD**	**LGD**	**ECL**	**Discount factor**	**Provision**
1	**£1,000,000**	**0.4%**	**30%**	**£1,200**	**0.952381**	**£1,142.86**
2	**£1,000,000**	**1.2%**	**50%**	**£6,000**	**0.907029**	**£5,442.18**
3	**£1,000,000**	**1.2%**	**50%**	**£6,000**	**0.863838**	**£5,183.02**
4	**£1,000,000**	**0.4%**	**30%**	**£1,200**	**0.822702**	**£987.24**
5	**£1,000,000**	**0.4%**	**30%**	**£1,200**	**0.783526**	**£940.23**
			Total provision for expected credit losses			**£13,695.53**

Strictly speaking, ABC Bank should have allowed for a declining EAD over the 5 years, consistent with its expectations for defaults. However, ABC took the view that, given the approximate nature of the PD and LGD estimates, a more accurate quantification of EADs, allowing for defaults, was not necessary.